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Fractional Quantum Hall Effect and (2 + 1)-Dimensional Quantum Electrodynamics

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By investigating the gap equation defining the full electron propagator in the framework of (2 + 1)-dimensional quantum electrodynamics, we derive the fractional quantum Hall effect at the filling factors $\nu = 1/2, 1/3, 2/5, 1/9$. In particular, the value $\nu = 1/2$ emerges as the lowest order of the development of this gap equation.

The fractional quantum Hall effect (FQHE) (Prange and Girvin, 1990) occurs in a two-dimensional electron gas (2DEG) in a strong magnetic field oriented perpendicular to the plan of the electrons. The effect is understood to be the result of an excitation gap in the spectrum of an infinite 2DEG at these magnetic fields. This effect is characterized by quantized rational values of the Hall resistivity,

$$\rho_{xy} = \frac{h}{\nu e^2}, \qquad \nu = 1/3,\ 2/3,\ 1/5,\ \ldots \tag{1}$$

where ν is the ratio of the number of particles to the number of avaible states in a magnetic subband (Landau level), or equivalently, the number of particles per flux quantum. In the FQHE, the excitation gap is a consequence of the strong electron–electron correlations.

Recently, at the qualitative level, the most dramatic discovery made so far in the study of double-layer quantum Hall systems was the observation of the even denominator $\nu = 1/2$ (Suen *et al.*, 1992; Eisentein *et al.*, 1992). In both cases, a deep minimum in the diagonal resistivity ρ_{xx} was found at total filling factor 1/2 along with a well-defined plateau in the Hall resistivity at

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$$\rho_{xy} = 2\,\frac{h}{e^2} \tag{2}$$

An alternative approach to understanding the behavior at $\nu = 1/2$ is suggested by the ideas of transmutability of statistics for particles in two-dimensional systems (Halprin, 1984a, b; Wilczek, 1990; Jain, 1989; Jain *et al.*, 1990; Zhang, 1992; Laughlin, 1988; Movre and Read, 1991). In particular, it is possible to introduce a Chern–Simons gauge field that interacts with the electrons, which is equivalent to attaching to each a "magnetic flux tube."

The purpose of this work is to use a theoretical approach which include a model of the field theory in the framework of the quantum electrodynamic (2 + 1)-dimensional space. Indeed, we consider the gap equation (Acharya and Swamy, 1994) derived from the Dyson–Schwinger equation defining the exact electron propagator, by expanding the gap equation in the region where the electron momentum p satisfies the condition $|p| < m$, with m defining the effective mass or the physical mass of the electron. In this case, we can show that the filling factor is equal to 1/2 just by considering the zeroth order of the gap equation expansion. The other filling factors arise if the expansion is extended to the next orders, i.e., 1/3, 2/5, and 1/9 in the second, fourth and sixth orders, respectively.

By using the ladder approximation in the Landau gauge, the gap equation for the self-energy of the electrons $\Sigma\,(p^2)$ in (2 + 1)-dimensional QED takes the form (Acharya and Swamy, 1994)

$$\Sigma(p^2) = -\frac{2ie^2}{(2\pi)^3}\int d^3k\,\frac{\Sigma(k^2)}{(p-k)^2[k^2 - \Sigma^2(p^2)]} \tag{3}$$

where e is the electron charge.

On the other hand, the Maris–Herscovitz–Jacob (MHJ) (Maris *et al.*, 1969) approximation, which is valid for low as well as high momentum, consists in replacing $\Sigma(-p^2) \simeq m$ only in the denominator of the integrand of (3). Then, we obtain

$$\Sigma(p^2) = -\frac{2ie^2}{(2\pi)^3}\int d^3k\,\frac{\Sigma(k^2)}{(p-k)^2[k^2 - m^2]} \tag{4}$$

In the Euclidean metric, equation (4) converts to

$$\Sigma(-p^2) = \frac{2e^2}{(2\pi)^3}\int d^3k\,\frac{\Sigma(-k^2)}{(p-k)^2[k^2 + m^2]} \tag{5}$$

By using the method of Fourier transform, it was shown by Acharya and Swamy (1994) that a solution of (5) is given by

$$\Sigma(-p^2) = \frac{m^3}{p^2 + m^2} \tag{6}$$

As mentioned before, in the case where the electron momentum satisfies the condition $|p| < m$, however, the expansion of the gap equation (6) can be written as

$$\Sigma(-p^2) = m \sum_{n=0}^{\infty} \left(\frac{-p^2}{m^2}\right)^n \tag{7}$$

On the other hand, the general expression for the filling factor in the Euclidean metric is given by (Acharya and Swamy, 1990)

$$\nu = \frac{1}{3\pi^2} \int d^3p \, \frac{3\Sigma(-p^2) + 2p^2 \, \Sigma'(-p^2)}{[p^2 + \Sigma^2(-p^2)]^2} \tag{8}$$

with $\Sigma'(-p^2)$ the derivative of $\Sigma(-p^2)$ with respect to $(-p^2)$.

We now insert the expansion (7) in (8) which gives the filling factor ν.

First, we consider the lowest order of the expansion given by (7), i.e.,

$$\Sigma(-p^2) \simeq m \tag{9}$$

From (8), we can derive the following integral, after putting $x = p/m$:

$$\nu = \frac{2}{\pi} \int_0^{\infty} dx \, \frac{x^2}{(1 + x^2)^2} \tag{10}$$

We obtain

$$\nu = \frac{1}{2} \tag{11}$$

This value defining the fractional quantum Hall effect at filling factor $\nu = 1/2$, which indicates that the Landau level is half-filled, has been discussed by many authors (Read, 1989; Greither *et al.*, 1991, 1992; Kivelson *et al.*, 1992; Schmeltzer, 1992). This exceptional value is an interesting feature of the FQHE, which has been known in terms of a fractional filling factor whose denominator is odd.

Second, we take the next order of this expansion,

$$\Sigma(-p^2) \simeq m - \frac{p^2}{m} \tag{12}$$

By using equation (12) in (8), we can write the latter equation as

$$\nu = \frac{-2}{3\pi} \int_0^{\infty} dx \frac{-3x^2 + x^4}{(1 - x^2 + x^4)^2} \tag{13}$$

We find

$$\nu = \frac{1}{3} \tag{14}$$

Third, we consider the fourth order of equation (7):

$$\Sigma(-p^2) \simeq m - \frac{p^2}{m} + \frac{p^4}{m^3} \tag{15}$$

From equations (15) and (8), the filling factor takes the following form:

$$\nu = \frac{-2}{3\pi} \int_0^{\infty} dx \frac{-3x^2 + x^4 + x^6}{(1 - x^2 + 3x^4 - 2x^6 + x^8)^2} \tag{16}$$

By evaluating this integral, we find

$$\nu = \frac{2}{5} \tag{17}$$

Fourth, we write equation (7) in the sixth order as

$$\Sigma(-p^2) \simeq m - \frac{p^2}{m} + \frac{p^4}{m^3} - \frac{p^6}{m^5} \tag{18}$$

In this case, equation (8) becomes

$$\nu = \frac{-2}{3\pi} \int_0^{\infty} dx \frac{-3x^2 + x^4 + x^6 - 3x^8}{(1 - x^2 + 3x^4 - 4x^6 + 3x^8 - 2x^{10} + x^{12})^2} \tag{19}$$

We find

$$\nu = \frac{1}{9} \tag{20}$$

The results 1/3, 2/5, 1/9 characterizing the fractional quantum Hall effect have been obtained by various methods (Frölhich and Zee, 1991; Jellal, 1997).

By using an expansion of the gap equation, we have shown that the fractional quantum Hall effect (FQHE) appears at the filling factors $\nu = 1/2, 1/3, 2/5, 1/9$. The most interesting case is $\nu = 1/2$, which is not a usual value for the FQHE. We believe that other filling factors can be given by considering other terms in the expansion of the gap equation.

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